Higher-order results in the electroweak theory

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Abstract. The present status of higher-order results in the electroweak theory is summarised, with particular emphasis on recent two-loop results for the prediction of the W-boson mass in the Standard Model and leading three-loop corrections to the rho parameter. The remaining theoretical uncertainties in the prediction for the W-boson mass and the effective weak mixing angle are discussed.

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1 Introduction

By comparing the experimental results for the electroweak precision observables, most prominently the W-boson mass, $M_{\rm W}$, and the effective weak mixing angle at the Z-boson resonance, $\sin^2 \theta_{\rm eff}$, with the predictions of the Standard Model (SM) and extensions of it, the electroweak theory can be tested at the quantum level. The current experimental errors in the determination of $M_{\rm W}$ and $\sin^2 \theta_{\rm eff}$ are $\delta M_{\rm W}^{\rm exp} = 34$ MeV and $\delta \sin^2 \theta_{\rm eff}^{\rm exp} = 0.00016$ [1], corresponding to a relative accuracy of 0.04% and 0.07%, respectively.

The prediction for $M_{\rm W}$ is obtained by using as input the Fermi constant measured in muon decay, G_{μ} , the Zboson mass, $M_{\rm Z}$, and the fine structure constant according to the relation

$$M_{\rm W}^2 \left(1 - \frac{M_{\rm W}^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} \left(1 + \Delta r \right),\tag{1}$$

where the quantity Δr summarises the radiative corrections. This is done by an iterative procedure, since Δr itself depends on $M_{\rm W}$, $\Delta r = \Delta r(M_{\rm W}, M_{\rm Z}, M_{\rm H}, m_{\rm t}, \ldots)$.

The effective weak mixing angle at the Z-boson resonance, $\sin^2 \theta_{\text{eff}}$, is defined by the effective vector and axial vector couplings for an on-shell Z boson,

$$\sin^2 \theta_{\rm eff} = \frac{1}{4} \left(1 - \frac{\operatorname{Re} g_V}{\operatorname{Re} g_A} \right).$$
 (2)

2 Higher-order results for $M_{ m W}$ and $\sin^2 heta_{ m eff}$

The one-loop result for Δr [2] can be written as

$$\Delta r^{(\alpha)} = \Delta \alpha - \frac{c_{\rm W}^2}{s_{\rm W}^2} \Delta \rho + \Delta r_{\rm rem}(M_{\rm H}), \qquad (3)$$

where $c_{\rm W}^2 = M_{\rm W}^2/M_{\rm Z}^2$, $s_{\rm W}^2 = 1 - c_{\rm W}^2$. It involves large fermionic contributions from the shift in the fine structure constant due to light fermions, $\Delta \alpha \propto \log m_f$, and from the leading contribution to the ρ parameter, $\Delta \rho$. The latter is quadratically dependent on the top-quark mass, $m_{\rm t}$, as a consequence of the large mass splitting in the isospin doublet [3]. The remainder part, $\Delta r_{\rm rem}$, contains in particular the dependence on the Higgs-boson mass, $M_{\rm H}$. Higher-order QCD corrections to Δr are known at $\mathcal{O}(\alpha \alpha_{\rm s})$ [4] and $\mathcal{O}(\alpha \alpha_{\rm s}^2)$ [5,6].

Recently the full electroweak two-loop result for Δr has been completed. It consists of the fermionic contribution [7,8,9], which involves diagrams with one or two closed fermion loops, and the purely bosonic two-loop contribution [10].

Beyond two-loop order the results for the pure fermionloop corrections (i.e. contributions containing *n* fermion loops at *n*-loop order) are known up to four-loop order [11]. They contain in particular the leading contributions in $\Delta \alpha$ and $\Delta \rho$. Most recently results for the leading three-loop contributions of $\mathcal{O}(G^3_{\mu}m^6_t)$ and $\mathcal{O}(G^2_{\mu}\alpha_sm^4_t)$ to the ρ parameter,

$$\Delta \rho^{(3)} = \frac{\Sigma_{\rm Z}^{(3)}(0)}{M_{\rm Z}^2} - \frac{\Sigma_{\rm W}^{(3)}(0)}{M_{\rm W}^2} \tag{4}$$

have been obtained for arbitrary values of $M_{\rm H}$ (by means of expansions around $M_{\rm H} = m_{\rm t}$ and for $M_{\rm H} \gg m_{\rm t}$) [12], generalising a previous result which was obtained in the limit $M_{\rm H} = 0$ [13]. In eq:delrho $\Sigma_Z^{(3)}(0)$ and $\Sigma_{\rm W}^{(3)}(0)$ denote the $\mathcal{O}(G^3_{\mu}m^6_{\rm t})$ and $\mathcal{O}(G^2_{\mu}\alpha_{\rm s}m^4_{\rm t})$ contributions to the transverse parts of the Z and W self-energies at vanishing external momentum. The corresponding shifts in $M_{\rm W}$ and $\sin^2 \theta_{\rm eff}$ are given by

$$\Delta M_{\rm W}^{(3)} \approx \frac{M_{\rm W}}{2} \frac{c_{\rm W}^2}{c_{\rm W}^2 - s_{\rm W}^2} \Delta \rho^{(3)},$$



Fig. 1. Shifts in $M_{\rm W}$, $\sin^2 \theta_{\rm eff}$ from the $\mathcal{O}(G^2_{\mu}m^6_{\rm t})$ (labelled " $X^3_{\rm t}$ contribution") and $\mathcal{O}(G^2_{\mu}\alpha_{\rm s}m^4_{\rm t})$ (labelled " $\alpha_{\rm s}X^2_{\rm t}$ contribution") contributions to $\Delta\rho$ (from [12])

$$\Delta \sin^2 \theta_{\rm eff}^{(3)} \approx -\frac{c_{\rm W}^2 s_{\rm W}^2}{c_{\rm W}^2 - s_{\rm W}^2} \Delta \rho^{(3)}.$$
 (5)

Their numerical effect is shown in Fig. 1. The $\mathcal{O}(G_{\mu}^{2}\alpha_{\rm s}m_{\rm t}^{4})$ contributions lead to a shift in $M_{\rm W}$ of up to 5 MeV and in $\sin^{2}\theta_{\rm eff}$ of up to 2.5×10^{-5} for $M_{\rm H} \lesssim 350$ GeV. The effect of the $\mathcal{O}(G_{\mu}^{3}m_{\rm t}^{6})$ contributions, on the other hand, is small. It does not exceed 1 MeV and 1×10^{-5} for $M_{\rm H} \lesssim 1$ TeV.

It does not exceed 1 MeV and 1×10^{-5} for $M_{\rm H} \lesssim 1$ TeV. While for $M_{\rm W}$ the complete electroweak two-loop result is known, the prediction for $\sin^2 \theta_{\rm eff}$ is currently based at the two-loop level on an expansion for large $m_{\rm t}$ up to the next-to-leading term of $\mathcal{O}(G_{\mu}^2 m_{\rm t}^2 M_Z^2)$ [14]. An evaluation of the complete two-loop contributions to $\sin^2 \theta_{\rm eff}$ is in progress [15].

3 Simple parametrisation of the full result for the W-boson mass

The full result for $M_{\rm W}$ containing all relevant corrections known so far is obtained from Δr given by

$$\Delta r = \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_{\rm s})} + \Delta r^{(\alpha\alpha_{\rm s}^2)} + \Delta r^{(\alpha^2)}_{\rm ferm} + \Delta r^{(\alpha^2)}_{\rm bos} + \Delta r^{(G^2_{\mu}\alpha_{\rm s}m_{\rm t}^4)} + \Delta r^{(G^3_{\mu}m_{\rm t}^6)}, \qquad (6)$$

where $\Delta r^{(\alpha)}$ is the one-loop result, eq:delrol, $\Delta r^{(\alpha\alpha_s)}$ and $\Delta r^{(\alpha\alpha_s^2)}$ are the two-loop [4] and three-loop [5,6] QCD corrections, and $\Delta r^{(\alpha^2)}_{\text{ferm}}$ [7,8,9] and $\Delta r^{(\alpha^2)}_{\text{bos}}$ [10] are the fermionic and purely bosonic electroweak two-loop corrections, respectively. The contributions $\Delta r^{(G^2_{\mu}\alpha_s m^4_t)}$ and $\Delta r^{(G^3_{\mu}m^6_t)}_{t}$ are obtained from the leading three-loop corrections to $\Delta \rho$ [12] specified in eq:delrho.

In eq:delrcontribs the pure fermion-loop contributions at three-loop and four-loop order obtained in floops are not included because their contribution turned out to be small as a consequence of accidental numerical cancellations, with a net effect of only about 1 MeV in $M_{\rm W}$ (using the real-pole definition of the gauge-boson masses). Since the result given in floops contains the leading contributions involving powers of $\Delta \alpha$ and $\Delta \rho$ beyond two-loop order, it is not necessary to make use of resummations of $\Delta \alpha$ and $\Delta \rho$ as it was often done in the literature in the past (see e.g. resum). Accordingly, the quantity Δr appears in eq:delr in fully expanded form.

In Table 1 the numerical values of the different contributions to Δr are given for $M_{\rm W} = 80.426$ GeV [1]. The other input parameters are [1]

$$m_{\rm t} = 174.3 \text{ GeV}, \ m_{\rm b} = 4.7 \text{ GeV}, M_{\rm Z} = 91.1875 \text{ GeV}, \ \Gamma_{\rm Z} = 2.4952 \text{ GeV}, \alpha^{-1} = 137.03599976, \ \Delta\alpha = 0.05907, \ \alpha_{\rm s}(M_{\rm Z}) = 0.119, G_{\mu} = 1.16637 \times 10^{-5} \text{ GeV}^{-2},$$
(7)

where $\Delta \alpha \equiv \Delta \alpha_{\text{lept}} + \Delta \alpha_{\text{had}}^{(5)}$. The total width of the Z boson, Γ_{Z} , appears as an input parameter since the experimental value of M_{Z} in eq:inputs, corresponding to a Breit– Wigner parametrisation with running width, needs to be transformed into the mass parameter defined according to the real part of the complex pole, which corresponds to a Breit–Wigner parametrisation with a constant decay width, see 2lfermb. It is understood that M_{W} in this paper always refers to the conventional definition according to a Breit–Wigner parametrisation with running width. The change of parametrisation is achieved with the one loop QCD corrected value of the W-boson width as described in 2lfermb.

Table 1 shows that the two-loop QCD correction,

 $\Delta r^{(\alpha \alpha_s)}$, and the fermionic electroweak two-loop correction, $\Delta r^{(\alpha^2)}_{\text{ferm}}$ are of similar size. They both amount to about 10% of the one-loop contribution, $\Delta r^{(\alpha)}$, entering with the same sign. The most important correction beyond these contributions is the three-loop QCD correction, $\Delta r^{(\alpha \alpha_s^2)}$, which leads to a shift in $M_{\rm W}$ of about -11 MeV. For large values of $M_{\rm H}$ also the contribution $\Delta r^{(G_{\mu}^2 \alpha_s m_t^4)}$ becomes sizable (see also the discussion of Fig. 1). The purely bosonic two-loop contribution, $\Delta r^{(\alpha^2)}_{\rm bos}$, and the leading electroweak three-loop correction,

 $\Delta r^{(G^3_{\mu}m^6_{\rm t})}$, give rise to shifts in $M_{\rm W}$ which are much smaller than even the experimental error envisaged for a future Linear Collider, $\delta M_{\rm W}^{\rm exp,LC} = 7$ MeV [18]. Since Δr is evaluated in Table 1 for a fixed value of $M_{\rm W}$, the contributions $\Delta r^{(\alpha\alpha_{\rm s})}$ and $\Delta r^{(\alpha\alpha_{\rm s}^2)}$ are $M_{\rm H}$ -in-

Since Δr is evaluated in Table 1 for a fixed value of $M_{\rm W}$, the contributions $\Delta r^{(\alpha\alpha_{\rm s})}$ and $\Delta r^{(\alpha\alpha_{\rm s}^2)}$ are $M_{\rm H}$ -independent. In the iterative procedure for evaluating $M_{\rm W}$ from Δr , on the other hand, also these contributions become $M_{\rm H}$ -dependent through the $M_{\rm H}$ -dependence of the inserted $M_{\rm W}$ value.

The electroweak two-loop result for $M_{\rm W}$ is very lengthy and involves numerical integrations of two-loop scalar integrals. It is therefore not possible to present the result for $M_{\rm W}$ in a compact analytic form. Instead, the full result for $M_{\rm W}$, incorporating all corrections listed in eq:delrcontribs, can be approximated by the following simple parametrisation [17],

$$M_{\rm W} = M_{\rm W}^0 - c_1 \,\mathrm{dH} - c_2 \,\mathrm{dH}^2 + c_3 \,\mathrm{dH}^4 + c_4 (\mathrm{dh} - 1)$$

Table 1. The numerical values (×10⁴) of the different contributions to Δr specified in tab:delrcontribs are given for different values of $M_{\rm H}$ and $M_{\rm W} = 80.426$ GeV (the W and Z masses have been transformed so as to correspond to the real part of the complex pole). The other input parameters are listed in eq:inputs (from mw2loop)

$M_{\rm H}/{ m GeV}$	$\Delta r^{(\alpha)}$	$\Delta r^{(\alpha \alpha_{\rm s})}$	$\Delta r^{(\alpha \alpha_{\rm s}^2)}$	$\Delta r_{\rm ferm}^{(\alpha^2)}$	$\Delta r_{\rm bos}^{(\alpha^2)}$	$\Delta r^{(G^2_\mu \alpha_{\rm s} m_{\rm t}^4)}$	$\Delta r^{(G^3_\mu m^6_{\rm t})}$
100	283.41	35.89	7.23	28.56	0.64	-1.27	-0.16
200	307.35	35.89	7.23	30.02	0.35	-2.11	-0.09
300	323.27	35.89	7.23	31.10	0.23	-2.77	-0.03
600	353.01	35.89	7.23	32.68	0.05	-4.10	-0.09
1000	376.27	35.89	7.23	32.36	-0.41	-5.04	-1.04

$$-c_{5} d\alpha + c_{6} dt - c_{7} dt^{2} - c_{8} dH dt + c_{9} dh dt -c_{10} d\alpha_{8} + c_{11} dZ, \qquad (8)$$

where

$$dH = \ln\left(\frac{M_{\rm H}}{100 \text{ GeV}}\right), \ dh = \left(\frac{M_{\rm H}}{100 \text{ GeV}}\right)^2,$$

$$dt = \left(\frac{m_{\rm t}}{174.3 \text{ GeV}}\right)^2 - 1, \ dZ = \frac{M_Z}{91.1875 \text{ GeV}} - 1,$$

$$d\alpha = \frac{\Delta\alpha}{0.05907} - 1, \ d\alpha_{\rm s} = \frac{\alpha_{\rm s}(M_Z)}{0.119} - 1,$$
(9)

and the coefficients $M_{\rm W}^0, c_1, \ldots, c_{11}$ take the following values (in GeV)

$$M_{\rm W}^0 = 80.3799, \ c_1 = 0.05429, \ c_2 = 0.008939, c_3 = 0.0000890, \ c_4 = 0.000161, \ c_5 = 1.070, c_6 = 0.5256, \ c_7 = 0.0678, \ c_8 = 0.00179, c_9 = 0.0000659, \ c_{10} = 0.0737, \ c_{11} = 114.9.$$
(10)

The parametrisation given in eq:fitformula–(10) approximates the full result for $M_{\rm W}$ to better than 0.5 MeV over the whole range of 10 GeV $\leq M_{\rm H} \leq 1$ TeV if all other experimental input values vary within their combined 2σ region around their central values given in eq:pardef. This should be sufficiently accurate for practical applications.

In view of the experimental exclusion bound on the Higgs-boson mass of $M_{\rm H} > 114.4$ GeV [19] it seems reasonable to restrict the Higgs-boson mass to the range 100 GeV $\leq M_{\rm H} \leq 1$ TeV. In this case a slight readjustment of the coefficients in eq:fitparams yields a parametrisation which approximates the full result for $M_{\rm W}$ even within 0.2 MeV, see mw2loop.

4 Remaining theoretical uncertainties

The theoretical predictions for the electroweak precision observables are affected by two kinds of uncertainties, namely the parametric uncertainty induced by the experimental errors of the input parameters, e.g. $m_{\rm t}$, and the uncertainty from unknown higher-order corrections.

The parametric uncertainties induced by varying the input values of $m_{\rm t}$, $M_{\rm Z}$, $\Delta \alpha_{\rm had}^{(5)}$ and $\alpha_{\rm s}(M_{\rm Z})$ by one standard deviation are shown for $M_{\rm W}$ and $\sin^2 \theta_{\rm eff}$ in Table 2.

Table 2. Approximate shifts in $M_{\rm W}$ and $\sin^2 \theta_{\rm eff}$ caused by varying the input parameters $m_{\rm t}$, $M_{\rm Z}$, $\Delta \alpha_{\rm had}^{(5)}$ and $\alpha_{\rm s}(M_{\rm Z})$ by 1σ around their experimental central values [1]

	$\delta M_{\rm W}/~{ m MeV}$	$\delta \sin^2 \theta_{\rm eff} / 10^{-5}$
$\delta m_{\rm t} = 5.1 \; {\rm GeV}$	31	-16
$\delta M_{\rm Z} = 2.1 \ {\rm MeV}$	2.6	1.4
$\delta\left(\Delta\alpha_{\rm had}^{(5)}\right) = 0.00036$	-6.5	13
$\delta \alpha_{\rm s}(M_{\rm Z}) = 0.0027$	-1.7	1.0

The dominant parametric uncertainty at present (besides the dependence on $M_{\rm H}$) is induced by the experimental error of the top-quark mass. It is about as large as the current experimental error for both $M_{\rm W}$ and $\sin^2 \theta_{\rm eff}$. The uncertainty caused by the experimental error of m_t will remain the dominant source of theoretical uncertainty in the prediction for $M_{\rm W}$ and $\sin^2 \theta_{\rm eff}$ even at the LHC, where the error on m_t will be reduced to $\delta m_t = 1-2$ GeV [20]. A further improvement of the parametric uncertainty of $M_{\rm W}$ will require the precise measurement of m_t at a future Linear Collider [21], where an accuracy of about $\delta m_t =$ 0.1 GeV will be achievable [18].

The second source of theoretical uncertainties in the prediction of the electroweak precision observables are the uncertainties from unknown higher-order corrections. Different approaches have been used in the literature for estimating the possible size of uncalculated higher-order corrections, see e.g. mwest,2lfermb. Since several of the corrections whose possible size had been estimated in the past have meanwhile been calculated, there exists some guidance concerning the reliability of the different methods. In mw2loop a careful analysis of the remaining uncertainties from unknown higher-order corrections in the prediction for $M_{\rm W}$ has been carried out. The three main sources of uncertainties in the prediction of $M_{\rm W}$ are from uncalculated corrections at $\mathcal{O}(G^2_{\mu}\alpha_{\rm s}m_{\rm t}^2M_Z^2), \mathcal{O}(G^3_{\mu}m_{\rm t}^4M_Z^2)$ and $\mathcal{O}(\alpha \alpha_s^3)$. The resulting theoretical uncertainty in the prediction for $M_{\rm W}$ has been estimated in mw2loop to be

$$\delta M_{\rm W}^{\rm theo} \approx 4 \,\,{\rm MeV}.$$
 (11)

This estimate holds for a relatively light Higgs boson, $M_{\rm H} \lesssim 300$ GeV. For a heavy Higgs boson, i.e. $M_{\rm H}$ close to the TeV scale, the remaining theoretical uncertainty is significantly larger.

While for the case of $M_{\rm W}$ unknown higher-order corrections are encountered only beyond the two-loop level, the prediction for $\sin^2 \theta_{\rm eff}$ is affected by further uncertainties arising from the non-leading fermionic two-loop contributions and the purely bosonic two-loop contributions, which have not yet been calculated. Using the same methods for estimating the theoretical uncertainties as in mw2loop, one finds for the remaining theoretical uncertainty in the prediction for $\sin^2 \theta_{\rm eff}$ from unknown higher-order corrections

$$\delta \sin^2 \theta_{\text{eff}}^{\text{theo}} \approx 6 \times 10^{-5}.$$
 (12)

The theoretical uncertainty of $\sin^2 \theta_{\text{eff}}$ is the dominant contribution to the "Blue Band" indicating the effect of the theoretical uncertainties from unknown higher-order corrections in the global SM fit to all data [1,23].

5 Comparison of the SM prediction for $M_{\rm W}$ with the experimental result

The theoretical prediction for $M_{\rm W}$ within the SM is shown as a function of the Higgs-boson mass in Fig. 2. The width of the band indicates the theoretical uncertainties, which contain the parametric uncertainties from varying the input parameters within one standard deviation (see Table 2) and the estimate of the uncertainties from unknown higher-order corrections given in eq:mwtheounc. As discussed above, the theoretical uncertainty is dominated by the effect of the experimental error of the top-quark mass.



Fig. 2. Prediction for $M_{\rm W}$ in the SM as a function of $M_{\rm H}$ for $m_{\rm t} = 174.3 \pm 5.1$ GeV. The current experimental value, $M_{\rm W}^{\rm exp} = 80.426 \pm 0.034$ GeV [1], and the experimental 95% C.L. lower bound on the Higgs-boson mass, $M_{\rm H} = 114.4$ GeV [19], are also indicated (from [17])

The theoretical prediction is compared in Fig. 2 with the current experimental value [1], taking into account the 95% exclusion bound from the direct search for the SM Higgs, $M_{\rm H} > 114.4$ GeV [19]. The comparison clearly favours a light Higgs-boson mass within the SM. Above the LEP exclusion bound on $M_{\rm H}$ the 1σ bands of the theory prediction and the experimental result for $M_{\rm W}$ overlap only in a small region, corresponding to $M_{\rm H}$ values significantly below 200 GeV.

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